

Magnetic Effect of Current & Magnetism

Topic Covered

- ☛ Oersted Experiment
- ☛ Lorentz Force
- ☛ Magnetic Force on a Current Carrying Conductor
- ☛ Motion in Magnetic Field
- ☛ Motion in Combined Electric and Magnetic Fields
- ☛ Force on a Current Carrying Conductor
- ☛ Biot-Savart's Law
- ☛ Application of Biot and Savart's Law
- ☛ Magnetic Field on the Axis of a Circular Current Loop
- ☛ Magnetic Field due to Straight Current Carrying Wire of Finite Length
- ☛ Magnetic Field Due to Infinitely Long current Carrying Wire
- ☛ Ampere's Law Force of Interaction Between parallel Wires
- ☛ Force & Torque on a current carrying Loop placed in uniform Magnetic Field
- ☛ Magnetic Behaviour of a Current Carrying Coil and its Magnetic Moment
- ☛ Current and Magnetic Field due to Circular Motion of a Charge

INTRODUCTION

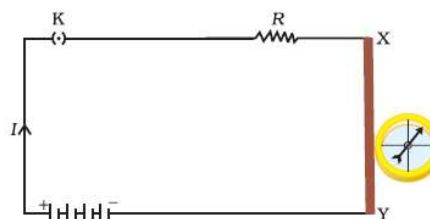
The term “**Magnetic Effects of Current**” means that “**the current flowing in a wire produces a magnetic field round it**”. The magnetic effect of current was discovered by **Oersted**. The electric motor, telephone and radio, all utilize the magnetic effect of current.

OERSTED EXPERIMENT

- An electric charge in motion setup a magnetic field around it.
- When a current flows through a wire, a nearby compass needle gets deflected.

This deflection of compass needle shows the presence of magnetic field.

- The phenomena of development of magnetic field around a wire, when an electric current flow through it is known as **Magnetic Effect of Current**.



Concept of Magnetic Field

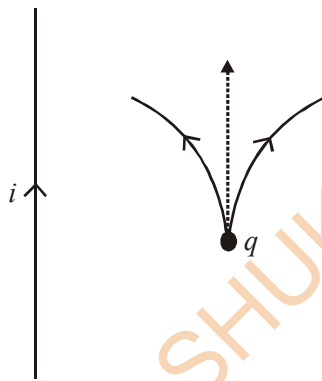
- (i) The region or space around current carrying conductor or magnet where magnetic effect can be felt.
- (ii) As electric field is associated with moving charge so is magnetic field with it.
- (iii) \vec{B} is the symbol of magnetic field.
- (iv) S.I. unit of magnetic field is tesla (T) OR Weber/ (meter)² OR $\text{NsC}^{-1}\text{m}^{-1}$

Dimensional formula for $B = [\text{MA}^{-1}\text{T}^{-2}]$

c.g.s. unit of magnetic field is Gauss (G) ; $1\text{G} = 10^{-4}\text{ T}$

If we place or suspend a small needle near a bar- magnet, the needle rests in a definite direction, If we place this needle at some other point, it rests in some other direction. This shows that the magnetic needle near the bar- magnet experiences a torque which turns the needle to a definite direction. ***The region near a magnet, where a 'magnetic needle experiences a torque and rests in a definite direction, is called magnetic field.'*** The line drawn from the south to the north pole of a magnetic needle freely suspended at a point in the magnetic field is the direction of the field at that point.

Suppose a charge 'q' starts moving with a velocity \vec{v} in a particular direction near a current carrying wire, Experimentally it is observed that the charge starts deflecting from its original path except a specific direction.

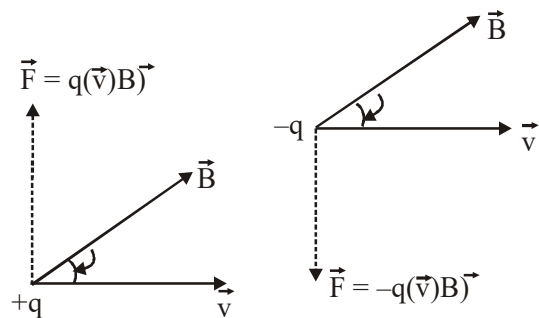


If \vec{v} is making an angle ' θ ' with this specific direction, a force \vec{F} is observed on it. Mathematically it can be expressed as

$$|\vec{F}| \propto v \sin \theta .$$

$|\vec{F}| \propto q; |\vec{F}| \propto \vec{B}$ (where \vec{B} is the intensity of magnetic field)

Direction of \vec{F} comes perpendicular to both, \vec{v} & the direction along which ' q ' does not deviate from its original path. The direction along which ' q ' does not deviate from its original path is known as direction of Magnetic field produced by the current carrying wire.



$$\vec{F} = q(\vec{v} \times \vec{B})$$

Direction of \vec{F} is given by right hand rule.

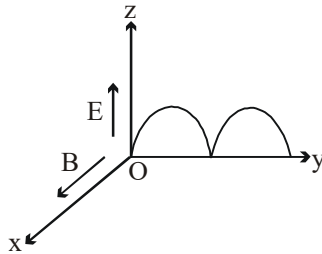
LORENTZ FORCE

Let us suppose that there is a point charge q (moving with a velocity \mathbf{v} and, located at \mathbf{r} at a given time t) in presence of both the electric field \mathbf{E} and the magnetic field \mathbf{B} . The force on an electric charge q due to both the fields can be written as

$$\text{i.e. } \vec{F} = \vec{F}_E + \vec{F}_B$$

$$= q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E} + \vec{v} \times \vec{B})$$

This force is called Lorentz force. It is a microscopic force.



If the particle starts moving from point O its path will be cycloid. (Unit of \vec{B} in M.K.S. system is weber/meter² or tesla or newton/ampere-meter.)

If a charge of 1 coulomb is moving in a direction perpendicular to the direction of magnetic field with a velocity of 1 m/s and one newton force acts on it, then the magnitude of magnetic intensity of field is 1 weber/meter².

☞ Since the Magnetic Force is always perpendicular to velocity so there is no work done by this force and hence the kinetic energy of the particle remains constant while moving in a magnetic region. That is speed of the particle remains constant.

MAGNETIC FORCE ON A CURRENT CARRYING CONDUCTOR

Suppose a conducting wire carrying a current i , is kept in a magnetic field \mathbf{B}

$$i = A n e V_d$$

Suppose $d\mathbf{l}$ be small element

where, $\mathbf{v}_d \rightarrow$ Drift velocity of free electrons

$A \rightarrow$ Area of cross-section of wire

$n \rightarrow$ number density of free electrons

$$\vec{F} = -e(\vec{v}_d \times \vec{B})$$

number of free electron in $d\mathbf{l} = nAd\mathbf{l}$

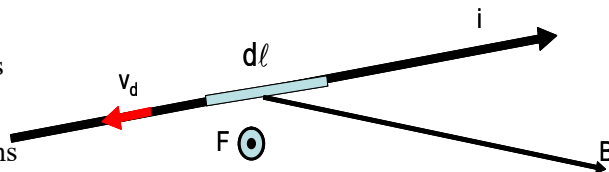
$$\text{magnetic field due to } d\mathbf{l} = d\vec{F} = (nAd\mathbf{l})(-e\vec{v}_d \times \vec{B})$$

$$d\vec{F} = i d\mathbf{l} \times \vec{B}$$

The quantity $i d\mathbf{l} \rightarrow$ current element

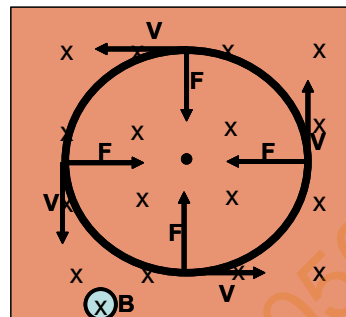
Total force on the wire of length l carrying a current i

$$\vec{F} = \vec{i}(l \times \vec{B})$$



MOTION IN MAGNETIC FIELD

- ⇒ The motion of a charged particle in uniform magnetic field is circular in nature (If it enters at 90° w.r.t \vec{B}).
- ⇒ The charged particle experiences a force $\vec{F}_m = q(\vec{v} \times \vec{B})$ changes its direction but its speed remains constant.
- ⇒ The nature of motion is helical in nature when charge enters at some angle say θ .



The radius of the circle described by the charged particle is

$$qvB = m \frac{v^2}{r}$$

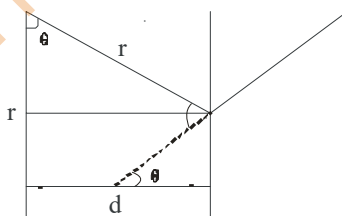
$$r = \frac{mv}{qB}$$

Time taken to complete the circle = T

$$T = \frac{2\pi r}{v}, \text{ put the value of } r,$$

$$T = \frac{2\pi m}{qB}$$

- ☞ If a charge enters a field region of insufficient length 'd', there is a deflection in its motion as shown in the figure.



$$\sin \theta = \frac{d}{r} \Rightarrow \theta = \sin^{-1} \left(\frac{d}{r} \right)$$

SOLVED EXAMPLES

Example 1. A test charge of 1.6×10^{-19} C is moving with velocity $\vec{v} = (2\hat{i} + 3\hat{j})$ m/s in a magnetic field $\vec{B} = (2\hat{i} + 3\hat{j})$ Wb/m². Find the force acting on the test charge.

Solution. $e = 1.6 \times 10^{-19}$ C; $\vec{v} = (2\hat{i} + 3\hat{j})$ m/s; $\vec{B} = (2\hat{i} + 3\hat{j})$ Wb m⁻²

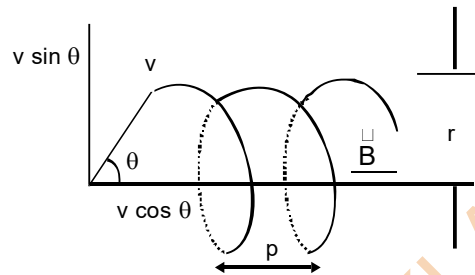
$$\vec{F} = e(\vec{v} \times \vec{B}) = 1.6 \times 10^{-19} [(2\hat{i} + 3\hat{j}) \times (2\hat{i} + 3\hat{j})] = 0$$

Example 2. An electron is projected with a velocity of 10^5 m/s at right angles to a magnetic field of 0.019 G. Calculate the radius of the circular path described by the electron, if $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg.

Solution. $\therefore v = 10^5$ m/s; $e = 1.6 \times 10^{-19}$ C; $m = 9.1 \times 10^{-31}$ kg; $B = 0.019$ G = 0.019×10^{-4} T

$$r = \frac{mv}{Be} = \frac{9.1 \times 10^{-31} \times 10^5}{0.019 \times 10^{-4} \times 1.6 \times 10^{-19}} = 0.299 \text{ m}$$

Example 3. A beam of protons with velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of helix. Mass of proton = 1.67×10^{-27} kg.



Solution. $r = \frac{mv \sin \theta}{qB}$ (\therefore component of velocity \perp to field is $v \sin \theta$)

$$= \frac{(1.67 \times 10^{-27})(4 \times 10^5)\sqrt{3}/2}{(1.6 \times 10^{-19})0.3} = \frac{2}{\sqrt{3}} \times 10^{-2} \text{ m} = 1.2 \text{ cm}$$

Again, pitch $p = v \cos \theta \times T$

$$\text{Where } T = \frac{2\pi r}{v \sin \theta}$$

$$\therefore p = \frac{v \cos \theta \times 2\pi r}{v \sin \theta} = \frac{\cos 60^\circ \times 2\pi \times (1.2 \times 10^{-2})}{\sin 60^\circ} = 4.35 \times 10^{-2} \text{ m} = 4.35 \text{ cm}$$

EXERCISE

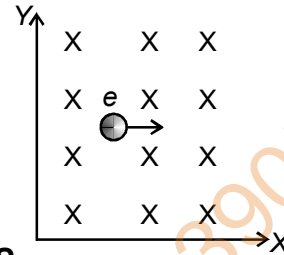
- If a particle of charge 10^{-12} coulomb moving along the \hat{x} -direction with a velocity 10^5 m/s experiences a force of 10^{-10} newton in \hat{y} -direction due to magnetic field, then the minimum magnetic field is
 - 6.25×10^3 tesla in \hat{z} -direction
 - 10^{-15} tesla in \hat{z} -direction
 - 6.25×10^3 tesla in \hat{z} -direction
 - 10^{-3} tesla in \hat{z} -direction
- An electron and a proton enter region of uniform magnetic field in a direction at right angles to the field with the same kinetic energy. They describe circular paths of radius r_e and r_p respectively. Then
 - $r_e = r_p$
 - $r_e < r_p$
 - $r_e > r_p$
 - r_e may be less than or greater than r_p depending on the direction of the magnetic field

3. An electron is moving on a circular path of radius r with speed v in a transverse magnetic field B . e/m for it will be

(A) $\frac{v}{Br}$ (B) $\frac{B}{rv}$ (C) Bvr (D) $\frac{vr}{B}$

4. In the given figure, the electron enters into the magnetic field. It deflects in direction

(A) +ve X direction
(B) -ve X direction
(C) +ve Y direction
(D) -ve Y direction



MOTION IN COMBINED ELECTRIC AND MAGNETIC FIELDS

\vec{F}_e depends on E

\vec{F}_m depends on \vec{B} & \vec{v}

If $v = 0$, $\vec{F}_m = 0$ and only \vec{F}_e acts.

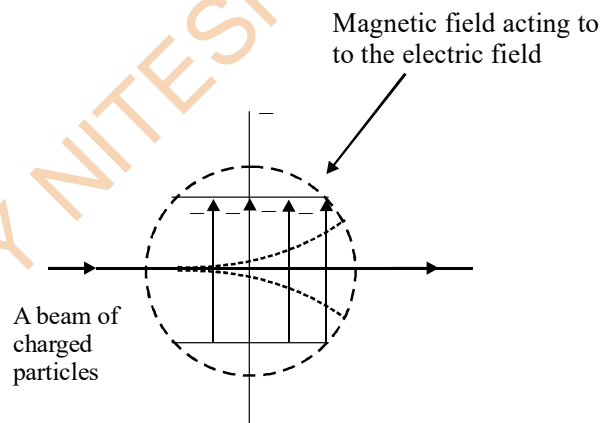
\vec{F}_m can only change the direction of motion of the charged particle

\vec{F}_m doesn't change the K.E. of the charged particle

\vec{F}_e changes the K.E. of the charged particle

Velocity Filter

Velocity filter is an arrangement of cross electric and magnetic fields which helps us to select from a beam, charged particles of the given velocity irrespective of their charge and mass.



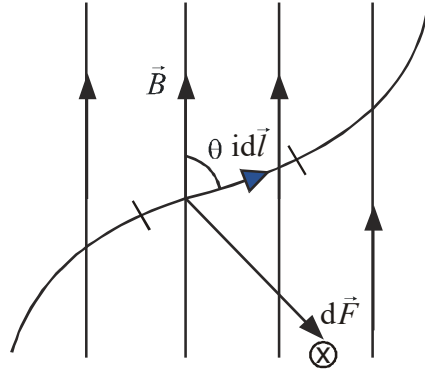
A velocity selector consists of two slits S_1 and S_2 held parallel to each other. In the region between the slits, uniform electric and magnetic fields are applied, perpendicular to each other as well as to the axis of slits. When a beam of charged particles of different charges and masses after passing through slit S_1 enters the region of crossed electric field E and magnetic field B , each particle experiences a force due to these fields. Those particles which are moving with the velocity v , irrespective of their mass and charge, if force on each particle due to electric field (qE) is equal and opposite to the force due to magnetic field ($q v B$), then

$$qE = q v B \text{ or } v = E/B$$

Such particles will go undeviated and filtered out of the region through the slit S_2 . Therefore, the particles emerging from slit S_2 will have the same velocity even though their charge and mass may be different.

FORCE ON A CURRENT CARRYING CONDUCTOR

Current is nothing but the flow of charges. Since flowing charges are in motion, they experience a magnetic force when kept in a magnetic field. The result is that the whole conductor experiences a force in the magnetic field.



Let dF be the Force acting on a small current element idl .

If a current 'i' is flowing into the wire, \vec{v}_d is the drift velocity of electrons opposite to the direction of current then.

Force on individual electron will be

$$\vec{f}_e = -e\vec{v}_d \times \vec{B}$$

So, the force on current elements \vec{idl} containing ' $nAdl$ ' electrons, where ' n ' is no. of electrons per unit volume and ' A ' is area of cross - section of the wire, is

$$\begin{aligned} d\vec{F} &= (nAdl)e\vec{v}_d \times \vec{B} \\ &= neA\vec{v}_d dl \times \vec{B} = \vec{idl} \times \vec{B} \end{aligned} \quad \dots(1)$$

If wire has a length l , then force on whole wire is $\vec{F} = i \vec{l} \times \vec{B} \dots(2)$

☞ The force experienced by an infinitesimal current element Idl placed in a magnetic field \vec{B} is given by

$$d\vec{F} = i d\vec{l} \times \vec{B}$$

The total force on a wire is the vector sum (integral) of the forces on all current elements

$$\int d\vec{F} = \int (i d\vec{l} \times \vec{B})$$

Thus force on a closed loop is always zero if \vec{B} is uniform.

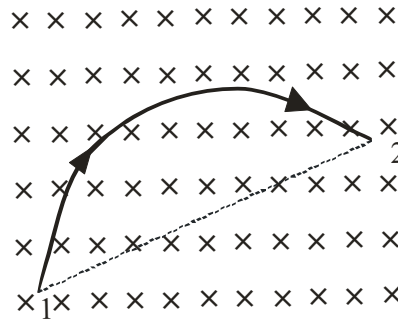
$$\vec{F} = \oint i d\vec{l} \times \vec{B} = \oint i d\vec{l} \times \vec{B} = 0$$

However different parts of the loop may experience elemental force due to which the loop may be in tension or may experience a torque.

In case the current loop is not closed and kept in a uniform field \vec{B}

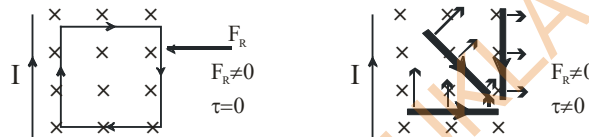
$$\vec{F} = \int d\vec{F} = i \int d\vec{l} \times \vec{B}$$

Here \vec{A} indicates the vector sum of all the individual elements. This can be interpreted as follows:



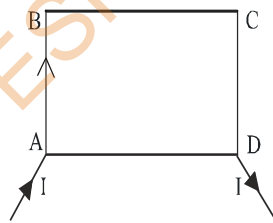
In the uniform field, the force on a curved wire joining points 1 and 2 is the same as the force on a straight wire joining these points, as shown in figure.

In a non-uniform field, resultant force on a loop or conductor is not zero. The torque τ may or may not be zero. If the conductor is free to move, it translates with or without rotation. In the figure below, two such situations are shown.



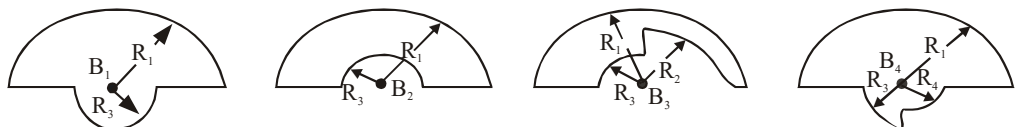
SOLVED EXAMPLES

Example 6. A uniform conducting wire forms a square loop ABCD. A current enters the loop at A and leaves at D. Find the magnetic field at the centre of the loop.



Solution. Obviously, the resistance along the path AB + BC + CD is three times the resistance of the parallel path AD. Therefore, the current I entering at A divides such that $1/4^{\text{th}}$ flows through ABCD and $3/4^{\text{th}}$ flows through AD. The combined magnetic field at the centre of the loop, produced due to the currents in AB, BC and CD will be equal and opposite to that due to the current in AD. Hence, the net field at the centre will be zero.

Example 7. In the four loops shown in the figure, all curved sections are either semicircles or quarter circles of radii R_1, R_2, R_3 and R_4 such that $R_1 > R_2 > R_3 > R_4$. All the four loops carry the same current. If B_1, B_2, B_3 and B_4 are the magnitude of the magnetic fields at the centres of the loops, then rank them according to magnitude, greatest first.



Solution.

When the two semicircles (or quarter circles) are on the opposite sides of the diameter (as in the first and fourth figure) the fields at the centre due to the two parts are in the same direction and hence add up. On the other hand, if the semicircles (or quarter circles) lie on the same side of the diameter (as in second and third figure), the fields due to the two parts are in opposite directions and hence get subtracted. Therefore, fields B_1 and B_4 will have greater magnitudes than the fields B_2 and B_3 .

As per Biot-Sarvat law, the magnitude of the magnetic field at the centre of a curved current-carrying elements is

$$dB = \left(\frac{\mu_0}{4\pi} \right) \frac{IdL}{R^2}$$

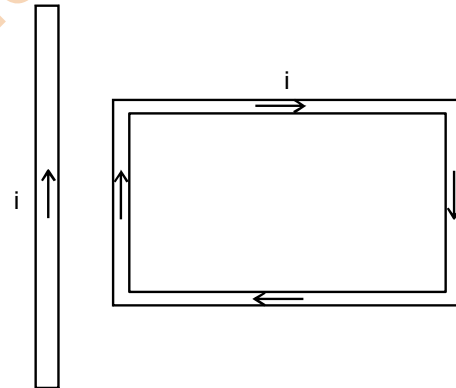
That is the magnitude of the fields is inversely proportional to R^2 . Since R_4 in fourth figure is less than R_3 in first figure, $B_4 > B_1$. Thus, B_4 is maximum.

In second figure, the field due to smaller semicircle is greater than that due to larger semicircle. The net field is the difference of the two fields. But, in third figure, the combined field due to the two quarter-circles will be less than that due to the smaller semicircle in second figure. Therefore, the field $B_3 < B_2$.

Thus, B_3 is the minimum, and $B_4 > B_1 > B_2 > B_3$.

EXERCISE

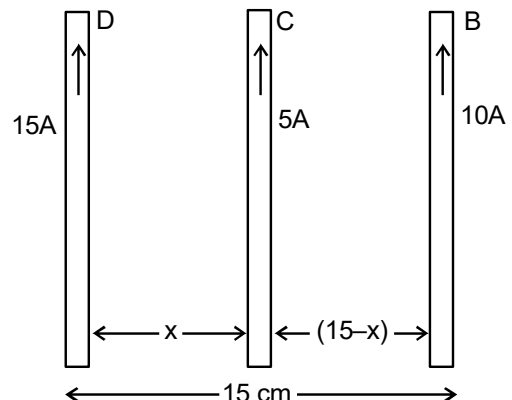
5. A rectangular loop carrying a current i is situated near a long straight wire such that the wire is parallel to the one of the sides of the loop and is in the plane of the loop. If a steady current I is established in wire as shown in figure, the loop will
- (A) Rotate about an axis parallel to the wire
 (B) Move away from the wire or towards right
 (C) Move towards the wire
 (D) Remain stationary



6. An electron moves with a constant speed v along a circle of radius r . Its magnetic moment will be (e is the electron's charge)

- (A) evr (B) $\frac{1}{2}evr$ (C) $\pi r^2 ev$ (D) $2\pi rev$

7. Three long, straight and parallel wires carrying currents are arranged as shown in the figure. The wire C which carries a current of 5.0 amp is so placed that it experiences no force. The distance of wire C from wire D is then
- (A) 9 cm
 (B) 7 cm
 (C) 5 cm
 (D) 3 cm

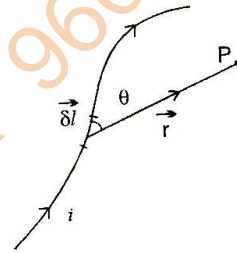


BIOT - SAVART'S LAW

- (i) The intensity of magnetic field due to current carrying conductor is determined with the help of this law.
- (ii) This law is obtained on the basis of experiments.
- (iii) The intensity of magnetic field (dB) at a certain point due to current carrying element of length (dl) of the conductor depends upon the following
 - (a) dB is directly proportional to the current flowing through the element, i.e. $dB \propto i$.
 - (b) dB is directly proportional to the length of the element of the conductor, i.e., $dB \propto dl$.
 - (c) dB is inversely proportional to the square of the distance r of the observation point P from the element, i.e., $dB \propto 1/r^2$
 - (d) dB is directly proportional to the sine of the angle θ between the position vector \vec{r} of the observation point P with respect to the element carrying element is given by
(in rationalized M.K.S. system)

$$\delta B = \frac{\mu_0}{4\pi} \frac{i \delta \ell \sin \theta}{r^2}$$

where μ_0 is magnetic permeability of vacuum and its value is $4\pi \times 10^{-7} = 12.57 \times 10^{-7}$ weber/ampere meter or henry/meter (H/m).



- (iv) In vector representation

$$\vec{\delta B} = \frac{\mu_0}{4\pi} \cdot \frac{i \vec{\delta \ell} \times \vec{r}}{r^3}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{i \vec{\delta \ell} \times \hat{r}}{r^2}$$

The direction of the element $\vec{\delta \ell}$ is taken in the direction of flow of current.

- (v) The direction of magnetic field is always perpendicular to the plane formed by the vectors $\vec{\delta \ell}$ and \vec{r}
- (vi) The intensity of magnetic field due to entire length of the conductor will be

$$B = \int \delta B = \int \frac{\mu_0}{4\pi} \cdot \frac{i \delta \ell \sin \theta}{r^2}$$

$$\text{or } B = \sum \delta B = \sum \frac{\mu_0}{4\pi} \cdot \frac{i \delta \ell \sin \theta}{r^2}$$

- (vii) If $\theta = 0^\circ$, that is the observation point is on the current carrying element,

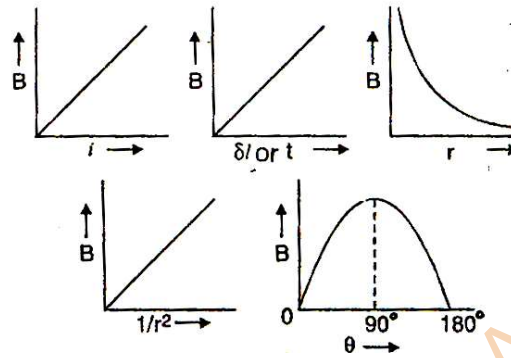
$$\therefore \quad dB = 0 \quad \text{or} \quad B = 0 \text{ (minimum)}$$

(viii) If the observation point is on the line perpendicular to the current carrying element, then

$$\theta = 90^\circ \quad \text{and} \quad \sin 90^\circ = 1$$

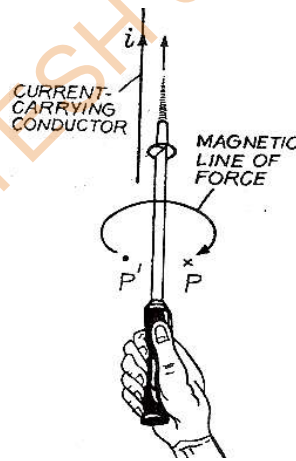
$$\therefore \delta B = \frac{\mu_0}{4\pi} \frac{i \delta \ell}{r^2} \quad (\text{maximum})$$

(ix) The graph between B and i, B and $\delta \ell$ or ℓ , B and r, B and $1/r^2$ and B and θ are as follows.

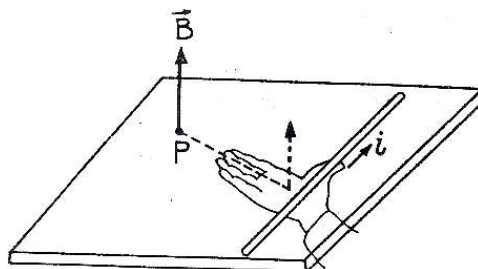


(x) **Direction of magnetic field :** The direction of magnetic field is determined with the help of the following simple laws :

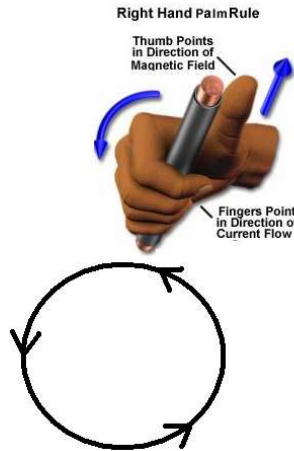
(a) **Maxwell's cork screw rule :** According to this law if a right handed cork screw is rotated in such a way that it moves forward in the direction of current in the conductor, then the direction of the rotation of the screw will show the direction of lines of force.



(b) **Right hand palm rule :** According to this rule if a current carrying conductor is held in the right hand such that the thumb of the hand represents the direction of current flow, then the direction of folding fingers will represent the direction of magnetic lines of force.



- (c) **Right hand palm rule for circular currents :** According to this rule if the direction of current in circular conducting coil is in the direction of folding fingers of right hand, then the direction of magnetic field will be in the direction of stretched thumb.



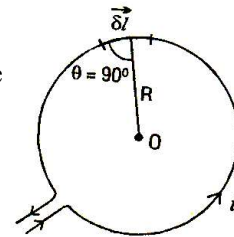
- (xi) The magnetic lines of force due to a current carrying element are in the form of concentric circles with their centres at the element.

APPLICATION OF BIOT AND SAVART'S LAW

Magnetic field due to a circular coil carrying current

- (i) If a circular coil of radius R is made from a wire of length L , then the number of turns in the coil will be

$$n = \frac{L}{2\pi R}$$



- (ii) If the current flowing in the coil is ' i ' amperes, then from Biot-Savart's law the magnetic field at the centre of current carrying coil is

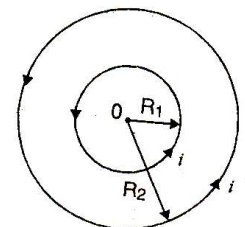
The direction of magnetic field is along the axis of the coil. If the current flowing in the coil is anticlockwise, then the direction of \vec{B}_0 will be upward along the axis of the coil. If the current is clockwise, then the direction of \vec{B}_0 will be inward along the axis of the coil.

If a coil of one turn is made from a wire of length L and another coil of n turns is made from a wire of same length and same amount of current flows through both coils, then the relation between the magnetic fields produced at their centres will be

$$|\vec{B}_{0n}| = n^2 |\vec{B}_{01}|$$

$$\text{or } \frac{|\vec{B}_{0n}|}{|\vec{B}_{01}|} = n^2$$

- (iii) If same amount of current is passed through two concentric and coplanar circular coils of radii R_1 and R_2 and number of turns n_1 and n_2 respectively, then the intensity of magnetic field at the centre will be



(a) when current flows in the same direction,

$$\vec{B}_0 = \vec{B}_{01} + \vec{B}_{02}$$

$$\text{or } B_0 = \frac{\mu_0 n_1 i}{2R_1} + \frac{\mu_0 n_2 i}{2R_2}$$

(b) when current flows in opposite directions,

$$B_0' = \frac{\mu_0 n_1 i}{2R_1} - \frac{\mu_0 n_2 i}{2R_2}$$

(iv) If any two points of a circular coil are connected to a battery and then current is passed through the coil. The magnetic field at the centre of the coil will be zero, i.e., $B_0 = 0$

(v) The resultant magnetic field at the centre of two perpendicular concentric circular current carrying coils is

$$|\vec{B}_0| = \sqrt{B_{01}^2 + B_{02}^2}$$

If B_0 makes an angle θ with B_{01} , then

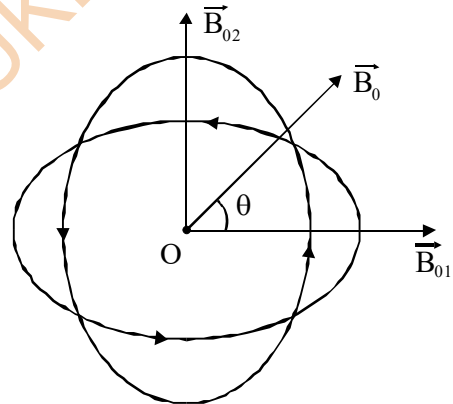
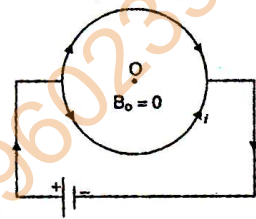
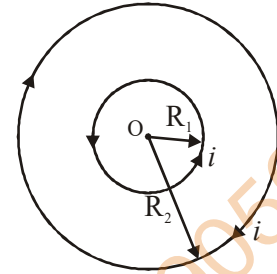
$$\tan \theta = \frac{B_{02}}{B_{01}}$$

$$\text{or } \theta = \tan^{-1} \left(\frac{B_{02}}{B_{01}} \right)$$

If the two coils are identical, then

$$B_0 = \sqrt{2} \left(\frac{\mu_0 n i}{2R} \right)$$

and $\theta = 45^\circ$



MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

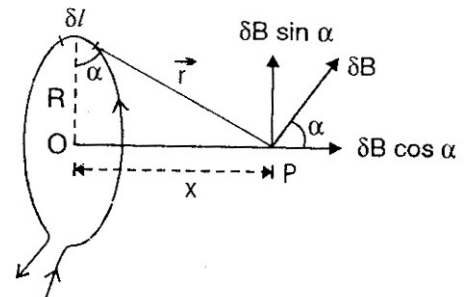
(vi) The magnetic field at an axial point situated at a distance

x from the centre of a current carrying circular coil:

In this position the observation point P is at the same distance r from each element of the coil where $r = \sqrt{R^2 + x^2}$

and the angle θ between the current element $\vec{\delta \ell}$ and the position vector \vec{r} is 90° . Hence $\sin \theta = 1$.

According to Biot-Savart's law the magnetic induction due to a current element is



$$\delta B = \frac{\mu_0}{4\pi} \frac{I \delta \ell}{r^2}$$

The components $\delta B \cos \alpha$ of magnetic induction $\vec{\delta B}$ due to different elements along the axis are in the same direction but the perpendicular components $\delta B \sin \alpha$ are symmetrically in different directions around the axis. Thus magnetic induction due to entire coil at the observation point, $\vec{B} = \Sigma \vec{\delta B} = \Sigma \delta B \cos \alpha$ along the axis and $\Sigma \delta B \sin \alpha = 0$ normal to the axis.

Thus $B = \Sigma \frac{\mu_0}{4\pi} \cdot \frac{l \delta \cos \alpha}{r^2}$ By putting $\cos \alpha$

$$= \frac{\mu_0 n i R^2}{2(R^2 + x^2)^{3/2}} \text{ along the axis}$$

or $B = \frac{\mu_0 n i R^2}{2R^3 (1 + x^2 / R^2)^{3/2}}$

$$= \frac{\mu_0 n i}{2R} \left(1 + \frac{x^2}{R^2}\right)^{-3/2} = B_0 \left(1 + \frac{x^2}{R^2}\right)^{-3/2}$$

If $x \gg R$, then

$$B = \frac{\mu_0 n i R^2}{2x^3}$$

$$\therefore B \propto \frac{1}{x^3}$$

(a) B depends upon the distance x. B decreases as x increases and $B = 0$ at $x = \infty$

(b) At a distance $\pm R$ from the centre of the coil

$$B = \frac{\mu_0 n i}{(4\sqrt{2})R} \quad \therefore \quad \frac{B}{B_0} = \frac{1}{2\sqrt{2}}$$

(c) when $x = \pm 0.766 R$, then $B = \frac{B_0}{2}$.

(d) B is not uniform near the coil. The rate of change of magnetic field with distance is different at different points.

At the centre of the coil, i.e. at $x = 0$

$$B_0 = \frac{\mu_0 n i}{2R}$$

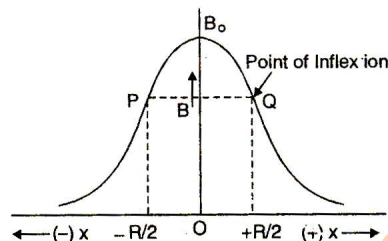
Thus the magnetic field at the centre of the coil is maximum.

(e) The graph between B and x is found to be of the following type.

- (f) The points of inflexion are those points on the curve where curvature becomes zero and the direction of curvature changes sign. The rate of change of magnetic field becomes constant at these points, i.e.,

$$\frac{dB}{dx} = \text{constant}$$

$$\text{or } \frac{d^2B}{dx^2} = 0.$$

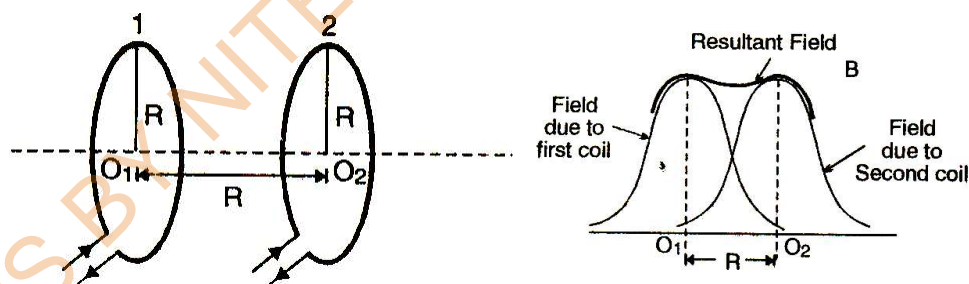


- (g) In the $(B - x)$ graph of current carrying coil two points of inflexion are found. They are at a distance $x = \pm R/2$ from the centre. The distance between the two points of inflexion is equal to the radius of the coil.

Helmholtz Coils

- (i) If two identical coils placed coaxially parallel to each other in such a way that the distance between them is equal to the radius of the coil and same amount of current is passed through them in the same direction, then in the middle region of the coil (near $x = \pm R/2$), on increasing the distance from the first coil field intensity will decrease but on decreasing the distance from the second coil the field intensity will increase by the same amount. Thus in this middle region the resultant magnetic field will be uniform. The two coils arranged in this way are called Helmholtz coils.
- (ii) These coils are used to produce uniform magnetic field.
- (iii) The uniform magnetic field produced in the middle region is

$$B = 2 \left[\frac{\mu_0 n i R^2}{2(R^2 + R^2/4)^{3/2}} \right] = \frac{8\mu_0 n i}{5\sqrt{5}R} = 0.716 \frac{\mu_0 n i}{R}$$



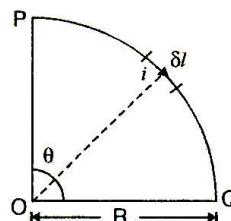
Magnetic Field due to a Current Carrying Arc

- (i) If an arc of radius R subtends an angle θ at its centre, then the length of the arc $= R\theta$. The magnetic field due to an element of the arc at the point O (from Biot-Savart's law)

$$\delta B_0 = \frac{\mu_0}{4\pi} \frac{i \delta l \sin \theta}{R^2}$$

$$\text{but } \theta = 90^\circ \quad \therefore \sin 90^\circ = 1$$

$$\text{Thus } \delta B_0 = \frac{\mu_0}{4\pi} \frac{i \delta l}{R^2}$$



The magnetic field due to entire arc at the point O

$$B_0 = \sum \delta B_0$$

$$= \sum \frac{\mu_0}{4\pi} \frac{i \delta l}{R^2} = \frac{\mu_0}{4\pi} \frac{i}{R^2} \sum \delta l$$

but $\sum \delta l = R\theta$ where θ is in radians

$$\therefore B_0 = \frac{\mu_0}{4\pi} \frac{i}{R^2} (R\theta) = \frac{\mu_0 i \theta}{4\pi R}$$

Thus $B_0 \propto i$, $B_0 \propto \theta$ and $B_0 \propto \frac{1}{R}$

(a) If length of the arc is one fourth of the coil, then

$$\theta = \pi/2$$

$$\therefore B_0 = \frac{\mu_0 i}{4\pi R} \left(\frac{\pi}{2} \right) = \frac{\mu_0 i}{8R}$$

(b) If length of the arc is half of the coil, then

$$\theta = \pi$$

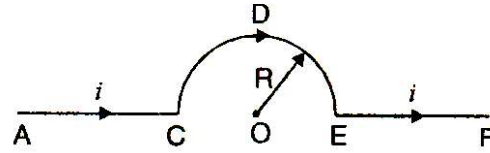
$$\therefore B_0 = \frac{\mu_0 i}{4\pi R} \pi = \frac{\mu_0 i}{4R}$$

(ii) Magnetic field due to current carrying conductor shown in the following figure. The magnetic field at O

$$B_0 = B_{AC} + B_{CDE} + B_{EF}$$

$$= 0 + \left(-\frac{\mu_0 i \pi}{4\pi R} \right) + 0$$

$$= -\frac{\mu_0 i}{4R}$$



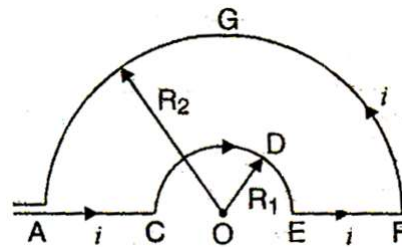
(Inward normal to the plane of the paper)

(iii) Magnetic field at the centre O of a current carrying conductor shown in the following figure

$$B_0 = B_{AC} + B_{CDE} + B_{EF} + B_{FGA}$$

$$= 0 + \left(-\frac{\mu_0 i}{4R_1} \right) + 0 + \left(\frac{\mu_0 i}{4R_2} \right)$$

$$= -\frac{\mu_0 i}{4} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) = -\frac{\mu_0 i}{4} \left(\frac{R_2 - R_1}{R_1 R_2} \right)$$



Negative sign shown that the magnetic field is inward normal to the plane of paper.

- (iv) The magnetic field at the centre O

$$B_0 = B_{AC} + B_{CDE} + B_{EF} + B_{FGA}$$

$$B_0 = 0 + \frac{\mu_0 i}{4R_1} + 0 + \frac{\mu_0 i}{4R_2}$$

$$\text{or } B_0 = \frac{\mu_0 i}{4} \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{\mu_0 i}{4} \left(\frac{R_1 + R_2}{R_1 R_2} \right)$$

Upward normal to the plane of paper.

- (v) The magnetic field at the centre O

$$B_0 = B_{CDE} + B_{AF}$$

$$= -\frac{\mu_0 i}{2R} + \frac{\mu_0 i}{2\pi R}$$

$$= -\frac{\mu_0 i}{2R} \left(1 - \frac{1}{\pi} \right)$$

Inward normal to the plane of paper.

- (vi) The magnetic field at the centre O

At point P the wires do not touch each other

$$B_0 = B_{AD} + B_{PQC}$$

$$= \frac{\mu_0 i}{2\pi R} + \frac{\mu_0 i}{2R} = \frac{\mu_0 i}{2R} \left(\frac{1}{\pi} + 1 \right)$$

Upward normal to the plane of paper.

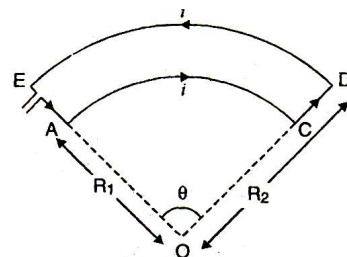
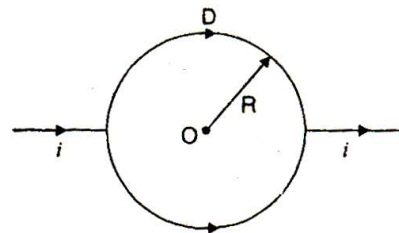
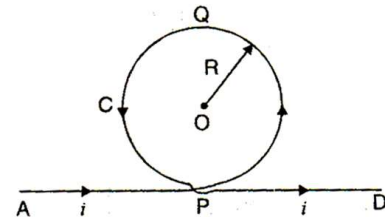
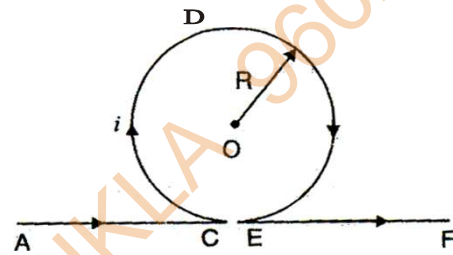
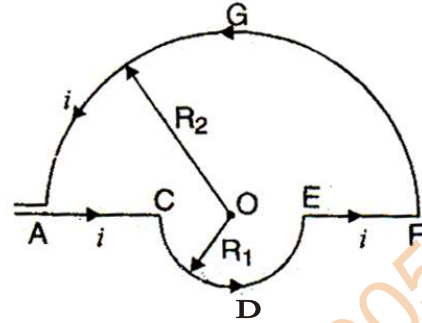
- (vii) The magnetic field at the centre O will be zero.

- (viii) The magnetic field at the O

$$B_0 = B_{AC} + B_{CD} + B_{DE} + B_{EA}$$

$$= -\frac{\mu_0 i \theta}{4\pi R_1} + 0 + \frac{\mu_0 i \theta}{4\pi R_2} + 0$$

$$= -\frac{\mu_0 i \theta}{4\pi} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$



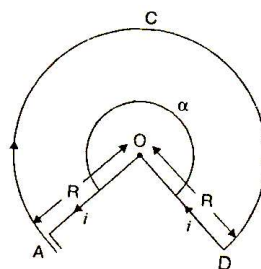
Negative sign shows that the magnetic field will be inward normal to the plane of paper.

- (ix) The magnetic field at the centre O

$$B_0 = B_{OA} + B_{ACD} + B_{DO}$$

$$= 0 - \frac{\mu_0 i \alpha}{4\pi R} + 0 = -\frac{\mu_0 i \alpha}{4\pi R}$$

Inward normal to the plane of paper.

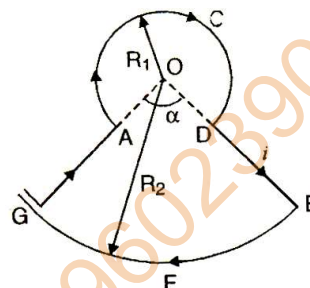


- (x) The magnetic field at the centre O

$$B_0 = B_{ACD} + B_{DE} + B_{EFG} + B_{GA}$$

$$= -\frac{\mu_0 i (2\pi - \alpha)}{4\pi R_1} + 0 + (-)\frac{\mu_0 i \alpha}{4\pi R_2} + 0$$

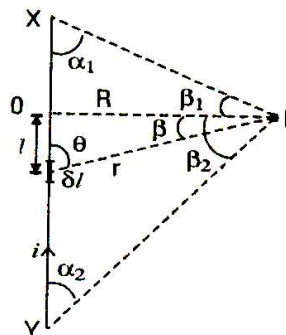
$$= -\frac{\mu_0 i}{4\pi} \left[\frac{2\pi - \alpha}{R_1} + \frac{\alpha}{R_2} \right] \text{ Inward normal to the plane of paper.}$$



MAGNETIC FIELD DUE TO STRAIGHT CURRENT CARRYING WIRE OF FINITE LENGTH

- (i) If a current 'i' is flowing through a conductor 'XY', then the magnetic field \vec{B} due to it at a point P can be determined using Biot-Savart's law. The normal distance of point P from the conductor is R. If the lines joining the ends X and Y of the conductor make angles β_1 and β_2 respectively with the perpendicular drawn on the conductor from the point P, then resultant magnetic induction at P due to current carrying conductor

$$B = \frac{\mu_0 i}{4\pi R} (\sin \beta_1 + \sin \beta_2)$$



If the lines joining the ends X and Y of the conductor with the point P make angles α_1 and α_2 respectively with the conductor, then

$$B = \frac{\mu_0 I}{4\pi R} (\cos \alpha_1 + \cos \alpha_2)$$

- (ii) If the current carrying conductor is of infinite length, then

$$\beta_1 = \beta_2 = \frac{\pi}{2}$$

$$B = \frac{\mu_0 i}{2\pi R} \text{ wb / m}^2$$

- (iii) The magnetic field at the centre of current carrying square coil of side a. If a current i is passed through a square coil ABCD of side a, then the magnetic field due to straight current carrying conductor AD of finite length

$$= \frac{\mu_0 i}{4\pi(a/2)} (\sin 45^\circ + \sin 45^\circ)$$

$$= \frac{\mu_0 i}{\sqrt{2}\pi a}$$

Due to symmetry the magnetic field due to each conductor will be $\frac{\mu_0 i}{\sqrt{2}\pi a}$. The direction of magnetic

field in this case will be $\frac{\mu_0 i}{\sqrt{2}\pi a}$. The direction of magnetic field in this case will be upward perpendicular to the plane of the coil.

∴ The magnetic field due to all the four sides of the square

$$B_0 = 4 \frac{\mu_0 i}{\sqrt{2}\pi a}$$

$$= 2\sqrt{2} \frac{\mu_0 i}{\pi a} \text{ tesla}$$

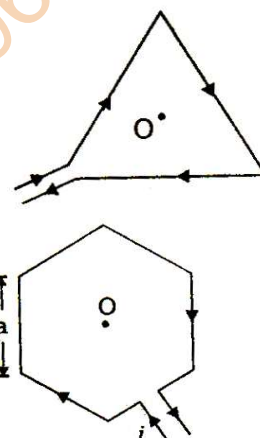
The direction of magnetic field will be normal to the plane of the coil.

- (iv) If a current i ampere is passed through the arms of an equilateral triangle of side a , then the magnetic field at its centre will be

$$B_0 = \frac{9\mu_0 i}{2\pi a}$$

- (v) If a current i ampere is passed in a hexagonal coil of side a . The magnetic field at the centre of the coil will be

$$B_0 = \frac{\mu_0 i \sqrt{3}}{\pi a}$$



MAGNETIC FIELD DUE TO INFINITELY LONG CURRENT CARRYING WIRE

- (i) Let current ' i ' be flowing in a long and straight wire. Wire is placed in the plane of paper. Magnetic field is determined at a distance r from this wire. The length of wire being very large relative to distance r , the wire can be assumed to be of infinite length. The magnetic lines of force due to current carrying wire will be in the form of concentric circles.

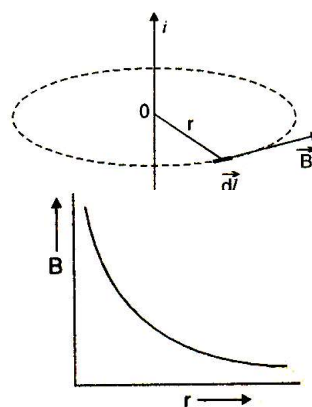
- (ii) If the magnetic induction at a distance r from the wire is B , then the circulation on the circular closed path of radius r .

$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = \mu_0 i$$

$$\therefore B = \frac{\mu_0 i}{2\pi r}$$

$$\text{or } B = \frac{\mu_0}{4\pi} \left(\frac{2i}{r} \right) = K \left(\frac{2i}{r} \right)$$

- (iii) The magnetic field \vec{B} due to long current carrying wire at a point near it (a) is proportional to the current i and (b) is inversely proportional to the distance r . This result can also be obtained from Bio-Savart's law.



(iv) The curve between B and r is as adjacent figure.

(v) The direction of magnetic field is perpendicular to the plane made by the wire and the position vector of the point.

Magnetic field due to two parallel and straight current carrying wires

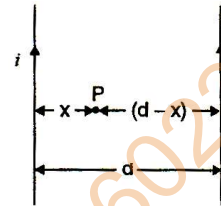
In the following figure, two long current carrying parallel wires are shown. The magnitude of current flowing in both wires is same. The distance between the wires is d. The magnetic field due to these wires at a point P will be.

(i) If the point P is in between the wires and

(a) currents in both wires are in the same direction

$$\vec{B}_p = \vec{B}_1 - \vec{B}_2$$

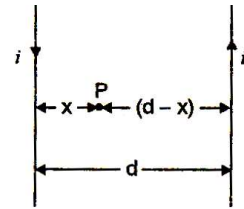
$$= \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} - \frac{1}{d-x} \right)$$



(b) currents in the two wires are in opposite directions

$$\vec{B}_p = \vec{B}_1 + \vec{B}_2$$

$$= \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} + \frac{1}{d-x} \right)$$

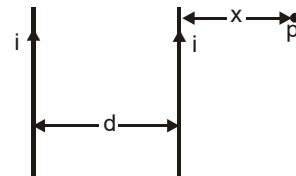


(ii) If the point P is outside the wire and

(a) currents in both wires are in the same direction

$$\vec{B}_p = \vec{B}_1 + \vec{B}_2$$

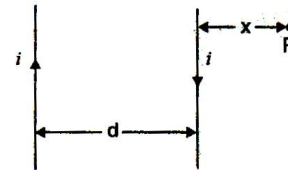
$$= \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} + \frac{1}{d+x} \right)$$



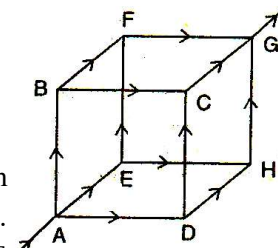
(b) current in the two wires are in opposite directions

$$\vec{B}_p = \vec{B}_1 - \vec{B}_2$$

$$= \frac{\mu_0 i}{2\pi} \left(\frac{1}{x} - \frac{1}{d+x} \right)$$



(vii) In a cubical structure made by wires if current is passed from one of its corner, then the magnetic field at its centre will be zero. Because AB and HG; AE and CG, AD and FG, BF and DH, FE and CD,



SOLVED EXAMPLE

Example 8. In an equilateral triangle of side 4.5×10^{-2} m, 1 ampere current is flowing. Find magnitude of B at its centroid.

Solution. Due to AB value of magnetic induction B_1 at O

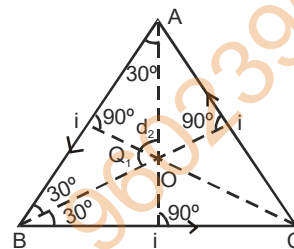
$$B_1 = \frac{\mu_0 i}{4\pi a} (\sin \phi_1 + \sin \phi_2)$$

here $i = 1$ A, $\phi_1 = \phi_2 = 60^\circ$

by figure $\frac{a}{\ell/2} = \tan 30$

$$\text{or } a = \frac{\ell}{2} \times \frac{1}{\sqrt{3}}$$

$$= \frac{4.5 \times 10^{-2}}{2\sqrt{3}}$$



At the centre of triangle O, $B = 3B_1$ and this will be upward perpendicular.

$$B = 3 \times \frac{4\pi \times 10^{-7} \times 1}{4\pi \times \frac{4.5 \times 10^{-2}}{2\sqrt{3}}} \times (\sin 60^\circ + \sin 60^\circ) = \frac{3 \times 10^{-7} \times 2\sqrt{3}}{4.5 \times 10^{-2}} \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right) = 4 \times 10^{-5} \text{ Wb/m}^2$$

Example 9. A current i is flowing in a square coil of side a , find the value of magnetic field at centre.

Solution. PQRS is current carrying square of side a due to which magnetic field at its centre O

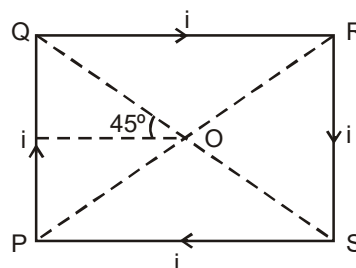
$$B = 4 \times B_{PQ}$$

$$= 4 \times \frac{\mu_0 i}{4\pi r} [\sin \theta_1 + \sin \theta_2]$$

here $\theta_1 = \theta_2 = 45^\circ$ and $r = \frac{a}{2}$

$$B = 4 \times \frac{\mu_0 i}{4\pi(a/2)} [\sin 45^\circ + \sin 45^\circ]$$

$$B = \frac{2\mu_0 i}{\pi a} \left(\frac{2}{\sqrt{2}} \right) \quad \text{or} \quad B = \frac{2\sqrt{2}\mu_0 i}{\pi a}$$

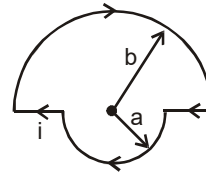


Example 10. You are given a closed circuit with radii a and b carrying current i .
The magnetic dipole moment of the circuit is

Solution. $m = \text{current} \times \text{area}$

$$= i \left(\frac{1}{2} \pi a^2 + \frac{1}{2} \pi b^2 \right)$$

$$= \frac{1}{2} i \pi (a^2 + b^2)$$

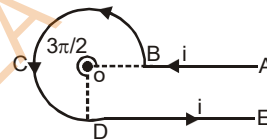


Example 11. In the body shown in diagram i current is flowing. Radius of circular part is r and linear parts are very long. Find magnitude of magnetic field at O .

Solution. $\vec{B}_0 = \vec{B}_{AB} + \vec{B}_{BCD} + \vec{B}_{DE}$

$$= 0 + \frac{3}{4} \left\{ \frac{\mu_0 i}{2r} \right\} \odot + \frac{1}{2} \left\{ \frac{\mu_0 i}{2\pi r} \right\} \odot$$

$$= \frac{\mu_0 i}{4\pi r} \left\{ \frac{3}{2} \pi + 1 \right\}$$



Example 12. If i_1 and i_2 currents are flowing in two wires mutually perpendicular to each other and placed very near, then find the equation of locus of the points of zero magnetic field.

Solution. $B_{PQ} = \left(\frac{\mu_0 i_1}{2\pi y} \right) \square$

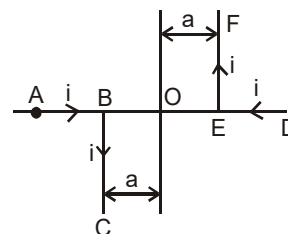
$B_{RS} = \left(\frac{\mu_0 i_2}{2\pi x} \right) \otimes$

For zero magnetic field $(B_{PQ}) \square = (B_{RS}) \otimes$

$$\text{or } \frac{\mu_0 i_1}{2\pi y} = \frac{\mu_0 i_2}{2\pi x} \quad \text{or } y = \left(\frac{i_1}{i_2} \right) x$$

Example 13. Two wire ABC and DEF are arranged as shown in figure. Current i is flowing through them. value of magnetic field at point O will be :

Solution. $B_0 = B_{AB} + B_{BC} + B_{DE} + B_{EF}$

$$= 0 + \frac{\mu_0 i}{4\pi a} + 0 + \frac{\mu_0 i}{4\pi a} = \frac{\mu_0 i}{2\pi a}$$


Example 14. A current of 2 ampere is made to flow through a coil which has only one turn. The magnetic field produced at the centre is $4\pi \times 10^{-6}$ Wb/m², then radius of the coil will be :

Solution. $B_0 = \frac{\mu_0 ni}{2R}$ (but $n = 1$)

$$B_0 = \frac{\mu_0 i}{2R} \quad \text{or} \quad R = \frac{\mu_0 i}{2B_0}$$

$$\text{or} \quad R = \frac{4\pi \times 10^{-7} \times 2}{2 \times 4\pi \times 10^{-6}} = 0.1 \text{ m}$$

Example 15. There are two concentric coils having same number of turns and radius 10 cm and 30 cm respectively. A current is made to flow in them (i) in same direction, (ii) in opposite direction. In both the cases the ratio of resultant magnetic fields at the centre of the coils will be :

Solution. (i) When the current flows in same direction

$$\begin{aligned} |\vec{B}_0| &= |\vec{B}_0 + \vec{B}_{02}| = \frac{\mu_0 ni}{2R_1} + \frac{\mu_0 ni}{2R_2} \\ &= \frac{\mu_0 ni}{2} \left(\frac{1}{0.1} + \frac{1}{0.3} \right) = \frac{\mu_0 ni}{2} \times \frac{40}{3} \end{aligned}$$

(ii) When the current flows in opposite direction

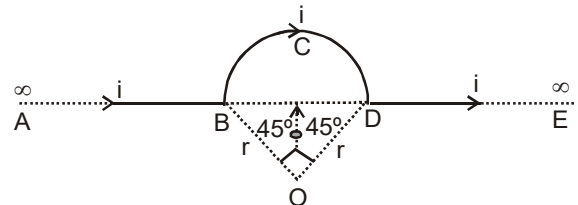
$$\begin{aligned} \vec{B}_0' &= \vec{B}_{01} - \vec{B}_{02} \\ B_0' &= \frac{\mu_0 ni}{2R_1} - \frac{\mu_0 ni}{2R_2} = \frac{\mu_0 ni}{2} \left[\frac{1}{0.1} - \frac{1}{0.3} \right] = \frac{\mu_0 ni}{2} \times \frac{20}{3} \\ \therefore \frac{B_0}{B_0'} &= \frac{2}{1} = 2:1 \end{aligned}$$

\therefore (2) is correct answer.

Example 16. In the given figure, find the magnetic field at the centre O :

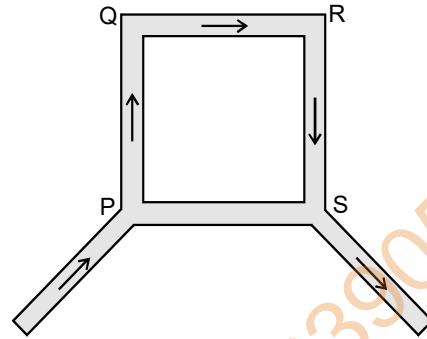
Solution. $B_0 = B_{AB} + B_{BCD} + B_{DE}$

$$\begin{aligned} &= \frac{\mu_0 i}{4\pi x} + \frac{1}{4} \frac{\mu_0 i}{2r} + \frac{\mu_0 i}{4\pi x} \\ &= 2 \frac{\mu_0 i}{4\pi x} + \frac{\mu_0}{4} \left(\frac{\pi i}{2r} \right) \\ &= \frac{\mu_0}{4\pi} \cdot \frac{2i\sqrt{2}}{r} + \frac{\mu_0}{4\pi} \left(\frac{\pi i}{2r} \right) \\ &= \frac{\mu_0}{4\pi} \left(\frac{2i}{r} \right) \left[\sqrt{2} + \frac{\pi}{4} \right] \end{aligned}$$



EXERCISE

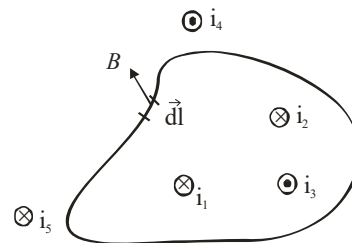
8. PQRS is a square loop made of uniform conducting wire. If the current enters the loop at P and leaves at S, then the magnetic field will be
- Maximum at the centre of the loop
 - Zero at the centre of loop
 - Zero at all points inside the loop
 - Zero at all points outside of the loop
9. Two infinitely long parallel wires carry equal current in same direction. The magnetic field at a mid point in between the two wires is
- twice the magnetic field produced due to each of the wires
 - half of the magnetic field produced due to each of the wires
 - square of the magnetic field produced due to each of the wires
 - zero



AMPERE'S LAW

Ampere's law gives another method to calculate the magnetic field due to a given current distribution. It is one of the four basic laws of electromagnetism. According to it, the circulation $\oint \vec{B} \cdot d\vec{l}$ of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. $d\vec{l}$ can have a direction of our choice. Sign of current on right hand side is given according to right hand rule, according to which if we curl fingers of our right hand along the chosen direction of $d\vec{l}$, currents in the direction of thumb will be taken as positive.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$



Where i is the sum of all out going and incoming currents $= i_3 - i_2 - i_1$ where we include the currents enclosed by the ampere's loop.

Applications of ampere's law

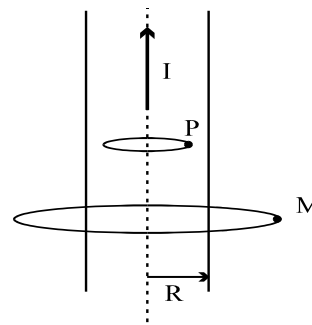
1. Magnetic field due to thick wire

(a) Inside Point

$$B \cdot 2\pi r = \mu_0 i \frac{\pi r^2}{\pi R^2} \Rightarrow B = \frac{\mu_0 i r}{2\pi R^2}$$

(b) Outside Point

$$B \cdot 2\pi r = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$$



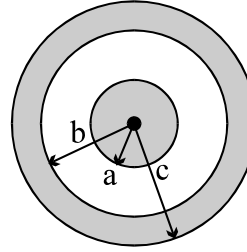
2. Magnetic field due to coaxial cable

$$B = \frac{\mu_0 i x}{2\pi a^2} \quad x < a$$

$$B = \frac{\mu_0 i}{2\pi x} \quad a \leq x \leq b$$

$$B = \frac{\mu_0 i (c^2 - x^2)}{2\pi x (c^2 - b^2)} \quad b < x < c$$

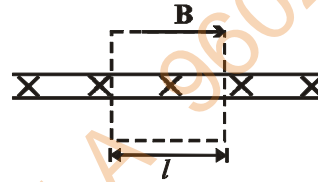
$$B = 0 \quad x \geq c$$



3. Magnetic field due to large metal sheet where Current per unit width of sheet = k

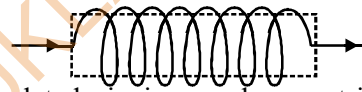
$$2Bl = \mu_0 kl$$

$$\Rightarrow B = \frac{1}{2} \mu_0 k$$



4. Magnetic field inside a solenoid

- Solenoid is a long cylindrical tube on which electrically insulated wire is wound symmetrically along its length.
- The diameter of a solenoid is small in comparison of its length and the plane of each turn of wire of an ideal solenoid can be considered to be normal to its axis.
- When current is passed through the solenoid, the magnetic behaviour at the points near each turn of the wire is similar to the straight current carrying wire and magnetic lines of force are in the form of concentric circles.
- The magnetic field \vec{B} at any point of the solenoid is equal to the vector sum of the magnetic fields produced by the different turns.
- The magnetic field inside a solenoid is almost uniform and along the axis and the magnetic field outside the solenoid at large distance is negligible in comparison to the magnetic field inside the solenoid thus the resultant magnetic field outside the solenoid at large distances can be considered to be zero.



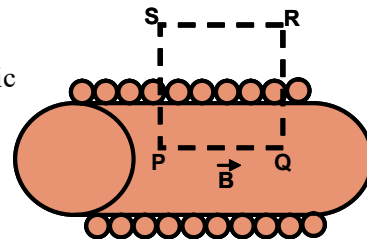
A solenoid of finite length : The magnetic field at a point P inside a current carrying solenoid of finite length

$$B_0 = \frac{\mu_0 n i}{2} (\cos \theta_1 - \cos \theta_2)$$

If L is the length of the solenoid and R its radius, then the magnetic field at its centre

$$B_{oc} = \frac{\mu_0 n i L}{\sqrt{4R^2 + L^2}}$$

where n is number of turns per unit length of solenoid.



Let I = Current through the solenoid.

n = number of turns per unit length

B = Magnitude of magnetic field inside the solenoid

$PQ = \ell$

$$\oint \vec{B} \cdot d\vec{\ell} = \int_P^Q \vec{B} \cdot d\vec{\ell} + \int_Q^R \vec{B} \cdot d\vec{\ell} + \int_R^S \vec{B} \cdot d\vec{\ell} + \int_S^P \vec{B} \cdot d\vec{\ell}$$

$$\int_P^Q \vec{B} \cdot d\vec{\ell} = \int_P^Q B d\ell \cos 0^\circ = B\ell$$

$$\int_Q^R \vec{B} \cdot d\vec{\ell} = \int_S^P \vec{B} \cdot d\vec{\ell} = 0 \quad (\because \text{angle b/w } \vec{B} \text{ \& } d\vec{\ell} = 90^\circ)$$

$$\int_R^S \vec{B} \cdot d\vec{\ell} = 0 \quad (\because \text{field outside the solenoid is } 0)$$

$$\oint \vec{B} \cdot d\vec{\ell} = \int_P^Q \vec{B} \cdot d\vec{\ell} = B\ell$$

According to Ampere's circuital law, we have

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \times [\text{Current enclosed by the loop PQRS}]$$

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \times \text{number of turns in PQRS} \times I$$

$$B\ell = \mu_0 \times n\ell \times I$$

$$B = \mu_0 n I$$

NOTE:

1. B depends on n & I and it is independent of the position within the solenoid. So, the field inside the solenoid is uniform.
2. If the solenoid is iron cored.

$$B = \mu_0 \mu_r n I = \mu n I \quad [\because \mu_0 \mu_r = \mu]$$

Where μ_r is the relative permeability of iron

3. Near ends of the solenoid

$$B = \frac{1}{2} \mu_0 n I$$

5. Magnetic field inside a toroid

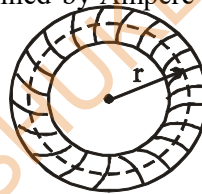
- (i) Toroid is like an endless cylindrical solenoid, i.e. if a long solenoid is bent round in the form of a closed ring, then it becomes a toroid.
- (ii) Electrically insulated wire is wound uniformly over the toroid as shown in the figure.
- (iii) The thickness of toroid is kept small in comparison to its radius and the number of turns is kept very large.
- (iv) When a current 'i' is passed through the toroid, each turn of the toroid produces a magnetic field along the axis at its centre. Due to uniform distribution of turns this magnetic field has same magnitude at their centres. Thus the magnetic lines of force inside the toroid are circular.
- (v) The magnetic field inside a toroid at all points is same but outside the toroid it is zero.
- (vi) If total number of turns in a toroid is N and R is its radius, then number of turns per unit length of the toroid will be

$$n = \frac{N}{2\pi R}$$

- (vii) The magnetic field due to toroid is determined by Ampere's law.

$$2\pi r B = \mu_0 N i$$

$$\Rightarrow B = \frac{\mu_0 N i}{2\pi r} = \mu_0 n i \quad (N = \text{total no. of turns})$$



6. Magnetic Field due to a Long Cylindrical Current Carrying Conductor

- (i) Current 'i' is flowing symmetrically through a straight long solid cylindrical conductor. Magnetic field is produced due to current flowing in the conductor around the conductor whose lines of force are concentric circles outside and inside the conductor.
- (ii) Magnetic field outside the conductor ($r > R$)

$$B_{\text{ext}} = \frac{\mu_0 i}{2\pi r}$$

Thus the magnetic field due to current carrying cylinder outside it is equivalent to the magnetic field due to thin current carrying wire.

- (iii) Magnetic field at the surface of the cylinder ($r = R$)

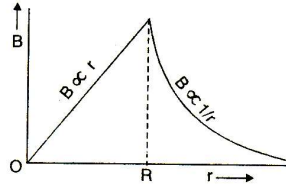
$$B_{\text{surface}} = \frac{\mu_0 i}{2\pi R}$$

- (iv) Magnetic field inside the conductor ($r < R$)

$$B_{\text{in}} = \frac{\mu_0 i}{2\pi R^2} r$$

Thus the magnetic field inside the cylindrical conductor is proportional to the distance r from the axis.

- (v) In the figure the dependence of magnetic field due to cylindrical conductor with the distance r from the axis is shown.



- (vi) **The magnetic field inside a current carrying pipe (hollow cylinder) :** If cylindrical conductor is hollow, i.e., it is in the form of pipe then the current flows through the surface only. In this case the current crossing an area πr^2 of a circular path inside the hollow part is zero. Thus from Ampere's law

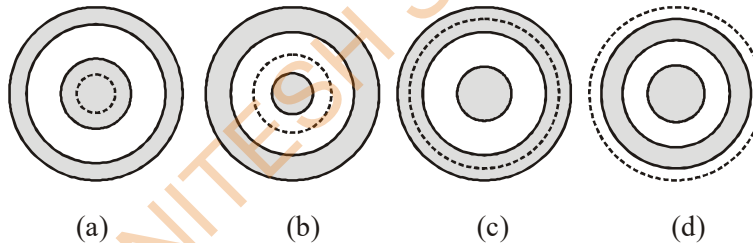
$$\oint \vec{B} \cdot d\vec{\ell} = B(2\pi r) = 0$$

$$B = 0$$

SOLVED EXAMPLE

- Example 17.** Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in opposite direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$ (c) $b < x < c$ and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Solution.



A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B} \cdot d\vec{\ell} = B 2\pi x \text{ in each of the four parts of the figure.}$$

- (a) The current enclosed within the circle in part a is

$$\frac{i_0}{\pi a^2} \cdot \pi x^2 = \frac{i_0}{a^2} x^2$$

Ampere's law $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 i$ gives

$$B 2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \text{ or, } B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B2\pi x = \mu_0 i_0 \quad \text{or,} \quad B = \frac{\mu_0 i_0}{2\pi x}$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$. Thus the current through this part is $\frac{i_0(x^2 - b^2)}{c^2 - b^2}$. This is in the opposite direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_0 - \frac{i_0(x^2 - b^2)}{c^2 - b^2} = \frac{i_0(c^2 - x^2)}{c^2 - b^2}$$

From Ampere's law,

$$B2\pi x = \frac{\mu_0 i_0 (c^2 - x^2)}{c^2 - b^2}$$

$$\text{or,} \quad B = \frac{\mu_0 i_0 (c^2 - x^2)}{2\pi x (c^2 - b^2)}$$

(d) The net current enclosed by the circle in part d of the figure is zero and hence

$$B2\pi x = 0 \quad \text{or,} \quad B = 0$$

FORCE OF INTERACTION BETWEEN PARALLEL WIRES

The interaction between two parallel wires can be summarised as follows :

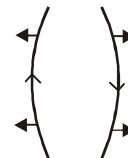
1. Like currents attract while unlike currents repel each other.
2. The force of interaction per unit length is proportional to the product of the currents in each wire.
3. The force is inversely proportional to the distance between them.

$$F = I_2 l_2 \times B_{21}$$

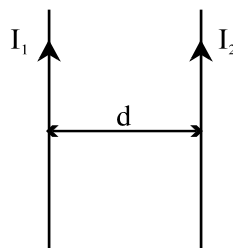
$$B_{21} = \frac{\mu_0 I_1}{2\pi d}$$



(a)



(b)



So
$$\frac{F}{l_2} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Note : Two parallel wires carrying current in the same direction attract each other while two similar charges moving parallel in the same direction repel each other. In first case only magnetic force acts while in second case both electric as well as magnetic forces act on the charges. Electric force (repulsive) is much higher than magnetic force. That is why charge beams of similar nature moving in same direction repel each other.

SOLVED EXAMPLE

Example 18. Two parallel wires are situated at a distance 10 cm and if 1 ampere current is flowing through each wire. The calculate the force per unit length on any one of them.

Solution. We know that

$$\begin{aligned}\frac{F}{\ell} &= \frac{\mu_0 i_1 i_2}{2\pi r} \\ &= \left(\frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 10 \times 10^{-2}} \right) \\ &= 2 \times 10^{-4} \text{ N/m}\end{aligned}$$

Example 19. As given in the figure X, Y and Z are three current carrying wires. What will be the direction of resultant magnetic force on wire Y ?

Solution. There will be attraction between X and Y and repulsion between X and Z. Both forces will act towards left \ Resultant force,

$$\begin{aligned}\vec{F}_{\text{Total}} &= \vec{F}_{YX} + \vec{F}_{YZ} \\ \vec{F}_{\text{Total}} &= \frac{\mu_0 I_x \times I_y}{2\pi r_1} + \frac{\mu_0 I_y \times I_z}{2\pi r_2}\end{aligned}$$

Example 20. A flexible conducting loop of wire of length 0.5 meter is kept in a magnetic field perpendicular to the plane of loop of value 1 tesla. Show that when the current flows in the loop then it acquires circular shape. Also calculate the tension in the wire if 1.57 amperes current is flowing through it.

Solution. Force on each small element dl of loop placed in magnetic field will be $iBdl$ and it will be perpendicular to the element dl

\therefore Loop will acquire circular shape

At equilibrium,

$$2T \sin \frac{\alpha}{2} = iBdl$$

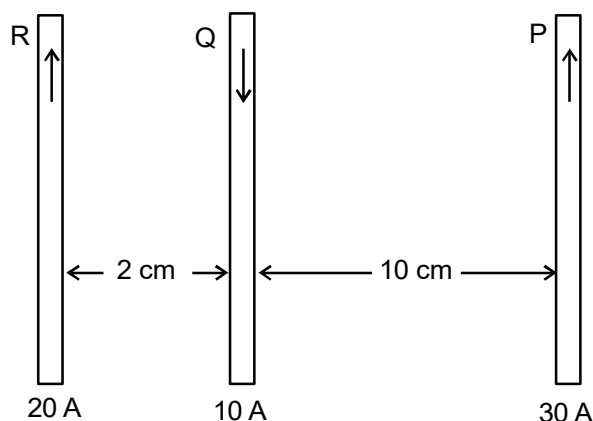
$$\text{If } \alpha \text{ is small } \sin \frac{\alpha}{2} \approx \frac{\alpha}{2}$$

$$\text{or } 2T \frac{\alpha}{2} = iBdl$$

$$\text{or } T = \frac{Bidl}{\alpha} \left\{ \text{but } \frac{dl}{\alpha} = r \right\} = Bir = \frac{iBl}{2\pi} = \frac{1.57 \times 1 \times 0.5}{2 \times 3.14} = 0.125 \text{ N}$$

EXERCISE

10. Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



- (A) 1.4×10^{-4} N towards right
(B) 1.4×10^{-4} N towards left
(C) 2.6×10^{-4} N towards right
(D) 2.6×10^{-4} N towards left

FORCE & TORQUE ON A CURRENT CARRYING LOOP PLACED IN UNIFORM MAGNETIC FIELD

- (i) A rectangular conducting loop PQRS is placed in a uniform magnetic field. Its length is 'l' and breadth is 'b'. The rotational axis of the loop is perpendicular to the magnetic field \vec{B} . When current i is passed through the loop, then the resultant force acting on the loop is zero but a torque acts on it.

- (ii) If plane of the loop makes an angle α with the direction of magnetic field, then the torque acting on the loop

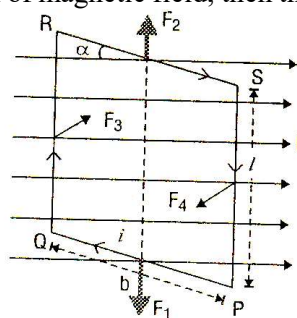
$$\tau = i/bB\cos\alpha$$

but $lb = A = \text{area of the rectangular loop}$

$$\therefore \tau = iBA\cos\alpha$$

If the loop has N turns, then

$$\tau = NiAB\cos\alpha$$



- (iii) If the vector area of the loop makes an angle θ with the direction of magnetic field, then $\theta = \frac{\pi}{2} - \alpha$

$$\therefore \text{Torque } \tau = iAB \sin\theta$$

For N turns $\tau = NiAB\sin\theta$

- (iv) $NiA = M = \text{magnetic moment of the loop}$

$$\therefore \tau = MB\sin\theta$$

In vector form, $\vec{\tau} = \vec{M} \times \vec{B}$

- (v) The torque acting on a current carrying loop placed in a magnetic field depends on its magnetic moment but not the shape of the loop.

- (vi) For maximum torque $\theta = 90^\circ$ or $\alpha = 0$, i.e., the plane or the loop should be in the direction of magnetic field or the vector area of the loop should be normal to the magnetic field.

$$\tau_{\max} = MB \sin 90^\circ = MB$$

- (vii) For minimum torque $\theta = 0^\circ$ or $\alpha = 90^\circ$, i.e., the plane of the loop should be normal to the magnetic field or vector area should be in the direction of magnetic field.

$$\tau_{\min} = MB \sin \theta = 0$$

- (viii) The torque acting on a current - carrying loop placed in a magnetic field depends on the current. Moving coil galvanometer is based on this principle.

- (ix) The work done in deflecting a coil from angle θ_1 to θ_2

$$W = MB(\cos \theta_1 - \cos \theta_2)$$

The work done in deflecting a coil from an angle 0 to θ in a magnetic field

$$W = MB(1 - \cos \theta).$$

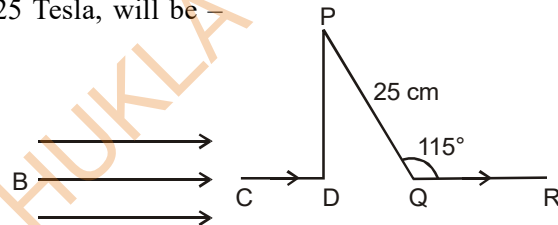
SOLVED EXAMPLES

Example 21. The magnetic force on segment PQ, due to a current of 5 amp. flowing in it, if it is placed in a magnetic field of 0.25 Tesla, will be –

Solution. $F = Bi/\sin \theta$

$$= 0.25 \times 5 \times 0.25 \sin 65^\circ$$

$$= 0.3125 \sin 65^\circ$$



Example 22. The length of a solenoid is 0.4 m and the number turns in it is 500. A current of 3 amp. is flowing in it. In a small coil of radius 0.01 m and number of turns 10, a current of 0.4 amp is flowing. The torque necessary to keep the axis of this coil perpendicular to the axis of solenoid will be –

Solution.

$$B_{\text{solenoid}} = \mu_0 n_s i_s$$

$$= \frac{\mu_0 N_s i_s}{L_s} \quad \text{or} \quad \tau = B_s i N A$$

$$= \frac{\mu_0 N_s i_s i N \pi r^2}{L_s}$$

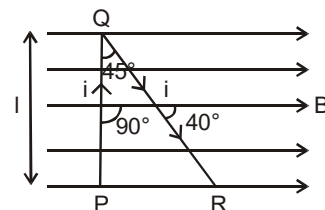
$$\tau = \frac{4\pi \times 10^{-7} \times 500 \times 3 \times 0.4 \times 10 \times \pi \times (0.01)^2}{0.4}$$

$$= 5.92 \times 10^{-6} \text{ N-m.}$$

Example 23. If i current is flowing in wire PQR. It is given the shape according to the figure and kept in a magnetic field B . If $PQ = l$ and $\angle PQR = 45^\circ$, then ratio of force on QR and PQ is :

Solution.

$$\frac{F_{QR}}{F_{PQ}} = \frac{Bil \sin 90^\circ}{Bi(l\sqrt{2}) \sin 45^\circ} = 1$$



Example 24. A conducting wire of length l is turned in the form of a circular coil and a current i is passed through it. For torque due to magnetic field produced at its centre, to be maximum, the number of turns in the coil will be–

Solution. $\tau_{\max} = MB$

$$\text{or } \tau_{\max} = ni\pi a^2 B$$

Let number of turns in length l is n

$$\ell = n(2\pi a) \quad \text{or} \quad a = \frac{\ell}{2\pi n}$$

$$\tau_{\max} = \frac{ni\pi B \ell^2}{4\pi^2 n^2} = \frac{\ell^2 i B}{4\pi n_{\min}}$$

$$\therefore \tau_{\max} \propto \frac{1}{n_{\min}}$$

$$n_{\min} = 1$$

Example 25. The effective radius of a coil of 100 turns is 0.05 m and a current of 0.1 amp is flowing in it. The work required to turn this coil in an external magnetic field of 1.5 Tesla through 180° will be, if initially the plane of the coil is normal to the magnetic field.

Solution. $W = 2MB$

$$W = 2\pi i N a^2 B$$

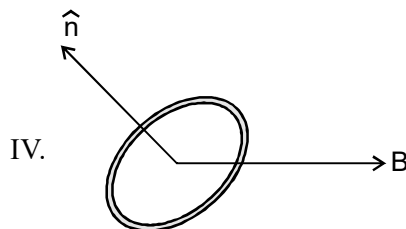
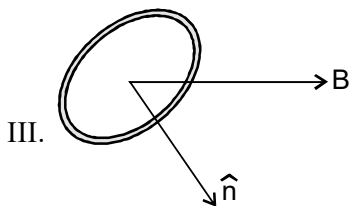
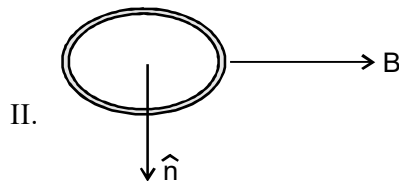
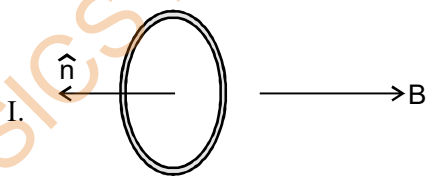
$$\text{or } W = 3.14 \times 2 \times 0.1 \times 100 \times (0.05)^2 \times 1.5 \\ = 0.236 \text{ Joule}$$

EXERCISE

11. A rectangular loop carrying a current i is placed in a uniform magnetic field B . The area enclosed by the loop is A . If there are n turns in the loop, the torque acting on the loop is given by

(A) $ni \vec{A} \times \vec{B}$ (B) $ni \vec{A} \cdot \vec{B}$ (C) $\frac{1}{n} (i \vec{A} \times \vec{B})$ (D) $\frac{1}{n} (i \vec{A} \cdot \vec{B})$

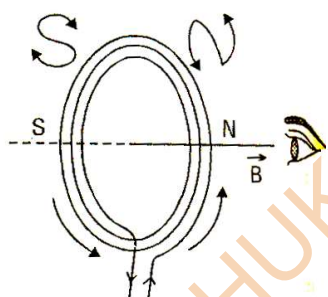
12. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III & IV. Arrange them in the decreasing order of potential energy



(A) $I > III > II > IV$ (B) $I > II > III > IV$ (C) $I > IV > II > III$ (D) $III > IV > I > II$

MAGNETIC BEHAVIOUR OF A CURRENT CARRYING COIL AND ITS MAGNETIC MOMENT

- (i) If current is passed in a small coil (loop), then the magnetic field due to it at a point on its axis is equivalent to the magnetic field due to a small magnet or magnetic dipole at a point on its axis.
- (ii) A current carrying small coil behaves like a small magnet.
- (iii) If the current in the coil is observed anticlockwise, then that plane of the coil behaves like the north pole.
- (iv) If the current in the coil is observed clockwise, then that plane of the coil behaves like the south pole.
- (v) Current carrying coil is equivalent to a magnetic strip.



- (vi) Magnetic moment of a current carrying coil is equal to the product of the current flowing in the coil and its effective area i.e.,

$$M = IA$$

- (vii) For a coil of one turn

$$M = IA = I\pi R^2$$

where R is the radius of a coil and I is the current flowing in the coil.

For a coil of N turns

$$M = NAI = N\pi R^2 I$$

- (viii) Unit of magnetic moment

Amp – m^2

ampere – m^2

- (ix) The magnetic field B_0 at the centre of a current carrying circular coil and its magnetic moment

M have the following relation. $B_0 = \frac{\mu_0 M}{2\pi R^3}$

- (x) If the magnetic moment of a current carrying small coil is M , then the magnetic field at a distance x ($x \gg R$) on its axis from the centre will be

$$B_p = \frac{\mu_0}{4\pi} \frac{2M}{x^3} = \frac{\mu_0 M}{2\pi x^3}$$

CURRENT AND MAGNETIC FIELD DUE TO CIRCULAR MOTION OF A CHARGE

- (i) According to the theory of atomic structure every atom is made of electrons, protons and neutrons. protons and neutrons are in the nucleus of each atom and electrons are assumed to be moving in different orbits around the nucleus.
- (ii) An electron and a proton present in the atom constitute an electric dipole at every moment but the direction of this dipole changes continuously and hence at any time the average dipole moment is zero. As a result static electric field is not observed.
- (iii) Moving charge produces magnetic field and the average value of this field in the atom is not zero.
- (iv) In an atom an electron moving in a circular path around the nucleus. Due to this motion current appears to be flowing in the electronic orbit and the orbit behaves like a current carrying coil. If e is the electron charge, R is the radius of the orbit and f is the frequency of motion of electron in the orbit, then

(a) current in the orbit = charge \times frequency = ef

If T is the period, then $f = \frac{1}{T}$

$$\therefore i = \frac{e}{T}$$

(b) Magnetic field at the nucleus (centre)

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ef}{2R}$$
$$= \frac{\mu_0 e}{2RT}$$

(c) If the angular velocity of the electron is ω , then

$$\omega = 2\pi f \quad \text{and} \quad f = \frac{\omega}{2\pi}$$

$$\therefore i = ef = \frac{e\omega}{2\pi}$$

$$\therefore B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 e\omega}{4\pi R}$$

(d) If the linear velocity of the electron is v , then

$$v = R\omega = R(2\pi f)$$

$$\text{or} \quad f = \left(\frac{v}{2\pi R} \right)$$

$$\therefore i = ef = \frac{ev}{2\pi R}$$

$$\therefore B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ev}{4\pi R^2}$$

- (v) Magnetic moment due to motion of electron in an orbit

$$M = iA = e f \pi R^2 = \frac{e \pi R^2}{T}$$

$$\text{or } M = \frac{e \omega \pi R^2}{2\pi} = \frac{e \omega R^2}{2} \quad \text{or} \quad M = \frac{e v \pi R^2}{2\pi R} = \frac{e v R}{2}$$

If the angular momentum of the electron is L , then

$$L = m v R = m \omega R^2$$

Writing M in terms of L

$$M = \frac{e m \omega R^2}{2m} = \frac{e m v R}{2m} = \frac{e L}{2m}$$

According to Bohr's second postulate

$$m v R = n \frac{h}{2\pi}$$

In ground state $n = 1$

$$L = \frac{h}{2\pi}$$

$$\therefore M = \frac{e h}{4\pi m}$$

- (vi) If a charge q (or a charged ring of charge q) is moving in a circular path of radius R with a frequency f or angular velocity ω , then

- (a) current due to moving charge

$$i = q f = q \omega / 2\pi$$

- (b) magnetic field at the centre of ring

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 q f}{2R} \quad \text{or} \quad B_0 = \frac{\mu_0 q \omega}{4\pi R}$$

- (c) magnetic moment

$$M = i(\pi R^2) \\ = q f \pi R^2 = \frac{1}{2} q \omega R^2$$

- (vii) If a charge q is distributed uniformly over the surface of plastic disc of radius R and it is rotated about its axis with an angular velocity ω , then

- (a) the magnetic field produced at its centre will be

$$B_0 = \frac{\mu_0 q \omega}{2\pi R}$$

- (b) the magnetic moment of the disc will be

$$M = \frac{q \omega R^2}{4}$$

SOLVED EXAMPLE

Example 26. A magnet having a magnetic moment of $1.0 \times 10^4 \text{ J/T}$ is free to rotate in a horizontal plane where a magnetic field $4 \times 10^{-5} \text{ T}$ exists. What is the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field?

- (A) 0.5 J (B) 4 J (C) 2 J (D) 0.2 J

Solution. (D)

Explanation:- Here, $M = 1.0 \times 10^4 \text{ J/T}$,

$$B = 4 \times 10^{-5} \text{ T}$$

$$W = ? \theta_1 = 0^\circ, \theta_2 = 60^\circ$$

$$\begin{aligned} W &= -MB (\cos\theta_2 - \cos\theta_1) \\ &= -1.0 \times 10^4 \times 4 \times 10^{-5} (\cos 60^\circ - \cos 0^\circ) \end{aligned}$$

$$= -0.4 \left(\frac{1}{2} - 1 \right) = 0.2 \text{ J}$$

Example 27. An electron is revolving around the nucleus of an atom in an orbit of radius 0.53 \AA . Calculate the equivalent magnetic moment if the frequency of revolution of electron is $6.8 \times 10^9 \text{ MHz}$.

Solution. Magnetic dipole moment, $p_m = IA$

$$A = \pi r^2$$

$$I = \frac{e}{T} = ef$$

$$f = 6.8 \times 10^9 \text{ MHz} = 6.8 \times 10^{15} \text{ Hz}$$

$$r = 0.53 \times 10^{-10} \text{ m}$$

$$p_m = ef \cdot \pi r^2$$

$$= 1.6 \times 10^{-19} \times 6.8 \times 10^{15} \times 3.14 \times (0.53 \times 10^{-10})^2$$

$$\therefore p_m = 9.6 \times 10^{-24} \text{ Am}^2$$

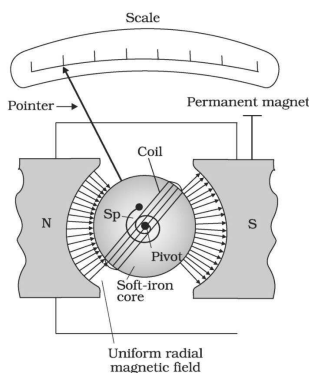
EXERCISE

- What happens to the speed and kinetic energy of a particle when I is projected perpendicular to the magnetic field.
- You have a coil of N turns, with current I through it. It is in a magnetic field B , which causes it to feel a torque. Give an expression for its magnetic dipole moment.

MOVING COIL GALVANOMETER

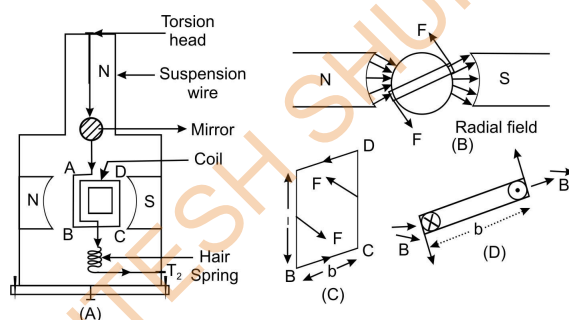
The current sensitivity of a moving coil galvanometer depends on,

- Number of turns in the galvanometer coil.
- Torsion constant of the suspension fiber.



The moving coil galvanometer. Its elements are described in the text. Depending on the requirement. This device can be used as a current detector or for measuring the value of the current (ammeter) or voltage (voltmeter).

(a) The labeled diagram of a moving coil galvanometer is given below.



Principle: It works on the principle that when a current carrying coil is suspended in a magnetic field then it experiences a torque.

Working: When current is passed along coil ABCD, the couple acts on it. Side AB of the coil experiences outward force and while CD experiences the inward force. The force experienced by coil is in accordance with Fleming's left hand rule. From the above figure it follows that the plane of coil always remains parallel to the magnetic field in all position of the coil (radial field). Because of this the force acts on the vertical arms of the coil in perpendicular direction always.

Let, I = The current flowing through coil.

B = magnetic field supposed to be uniform and always parallel to the coil

l = length of the coil

b = breadth of the coil

Deflecting torque acting on the coil is given by,

$$\tau = NI l b B \sin 90^\circ = NIAB$$

Where A = Area of the coil = $l \times b$

Due to deflecting torque, the coil rotates and suspension wire gets twisted. A restoring torque is set up in the suspension fiber. If θ is a angle through which the coil rotates and k is the restoring torque per unit angular twist, then Restoring torque, $\tau = k \theta$

At equilibrium position,

Deflecting torque = Restoring torque

$$\therefore NIBA = K\theta$$

$$\text{or } I = \left(\frac{k}{NBA} \right) \cdot \theta = G\theta.$$

Where $G = k/NBA$, is the galvanometer constant.

Hence it follows that,

$$I \propto \theta$$

From the above relation the linear scale for galvanometer can be set.

SOLVED EXAMPLE

Example 28. A galvanometer can be converted into an ammeter by using

- (A) low resistance in series (B) low resistance in parallel
(C) high resistance in series (D) high resistance in parallel

Solution. (B)

By using low resistance in parallel to a galvanometer; it can be converted into an Ammeter. The purpose of an ammeter is to show the amount of current passing through the circuit. Therefore maximum current is required to flow through the conductor. In order to do that, low resistance in parallel is to be connected.

Example 29. Why a galvanometer cannot be used as an ammeter directly?

Solution. The galvanometer cannot be used as an ammeter directly because for two reasons:

- Galvanometer is a very sensitive device, it gives a full scale deflection for a current of the order of mA and
- For measuring currents, the galvanometer has to be connected in series, and as it has a large resistance, this will change the value of current in the circuit

Example 30. In an ammeter, 0.5% of main current pass through galvanometer. If resistance of galvanometer is G , what will be the resistance of ammeter?

Solution. Here $I_g = 0.5\% \text{ of } I = 0.005I$

$$I_s = I - I_g = I - 0.005 I = 0.995 I$$

$$\therefore S = \frac{I_g G}{(I - I_g)} = \frac{0.005I \times G}{0.995 I} = \frac{G}{199}$$

EXERCISE

15. Explain the construction and working of a moving coil galvanometer. A moving coil galvanometer is made of a coil of radius 10cm and has 1000 turns. The strong horse shoe magnets create a magnetic field of 0.2T and the spring has a spring constant of 0.5 N/m . Find

(a) Galvanometer constant G

(b) Current sensitivity I_s

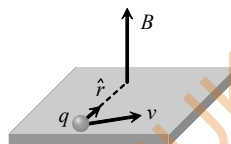
(c) Voltage Sensitivity V_s

Explain how a galvanometer can be made more sensitive

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Important Competition Tips

1. The device whose working principle based on Halmholtz coils and in which uniform magnetic field is used called as "Halmholtz galvanometer".
2. The value of magnetic field induction at a point, on the centre of separation of two linear parallel conductors carrying equal currents in the same direction is zero.
3. If a current carrying circular loop ($n = 1$) is turned into a coil having n identical turns then magnetic field at the centre of the coil becomes n^2 times the previous field i.e. $B_{(n \text{ turn})} = n^2 B_{(\text{single turn})}$
4. When a current carrying coil is suspended freely in earth's magnetic field, it's plane stays in **East-West** direction.
5. Magnetic field (\vec{B}) produced by a moving charge q is given by $\vec{B} = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \vec{r})}{r^3} = \frac{\mu_0}{4\pi} \frac{q(\vec{v} \times \hat{r})}{r^2}$; where v = velocity of charge and $v \ll c$ (speed of light).



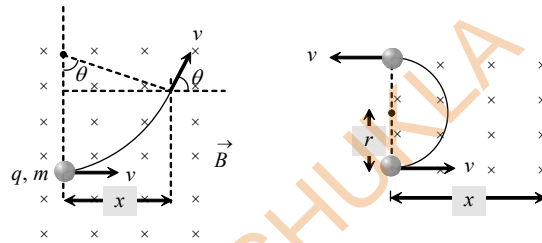
6. If an electron is revolving in a circular path of radius r with speed v then magnetic field produced at the centre of circular path $B = \frac{\mu_0}{4\pi} \cdot \frac{ev}{r^2} \Rightarrow r \propto \sqrt{\frac{v}{B}}$
7. The line integral of magnetising field (\vec{H}) for any closed path called magnetomotive force (MMF). It's S.I. unit is amp.
8. Ratio of dimension of e.m.f. to MMF is equal to the dimension of resistance.
9. The positive ions are produced in the gap between the two dees by the ionisation of the gas. To produce proton, hydrogen gas is used; while for producing alpha-particles, helium gas is used.
10. Cyclotron frequency is also known as magnetic resonance frequency.
11. Cyclotron can not accelerate electrons because they have very small mass.
12. The energy of a charged particle moving in a uniform magnetic field does not change because it experiences a force in a direction, perpendicular to it's direction of motion. Due to which the speed of charged particle remains unchanged and hence it's K.E. remains same.
13. Magnetic force does no work when the charged particle is displaced while electric force does work in displacing the charged particle.
14. Magnetic force is velocity dependent, while electric force is independent of the state of rest or motion of the charged particle.

15. If a particle enters a magnetic field normally to the magnetic field, then it starts moving in a circular orbit. The point at which it enters the magnetic field lies on the circumference. (Most of us confuse it with the centre of the orbit).

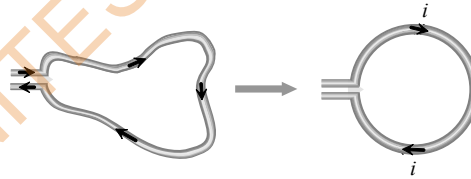
16. Deviation of charged particle in magnetic field : If a charged particle (q, m) enters a uniform magnetic field \vec{B} (extends upto a length x) at right angles with speed v as shown in figure. The speed of the particle in magnetic field does not change. But it gets deviated in the magnetic field.

17. Deviation in terms of time t ; $\theta = \omega t = \left(\frac{Bq}{m}\right)t$

18. Deviation in terms of length of the magnetic field; $\theta = \sin^{-1}\left(\frac{x}{r}\right)$. This relation can be used only when $x \leq r$. For $x > r$, the deviation will be 180° as shown in the following figure

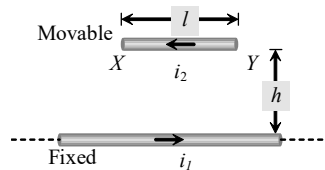


19. If no magnetic field is present, the loop will still open into a circle as in it's adjacent parts current will be in opposite direction and opposite currents repel each other.



20. In the following case if wire XY is slightly displaced from its equilibrium position, it executes

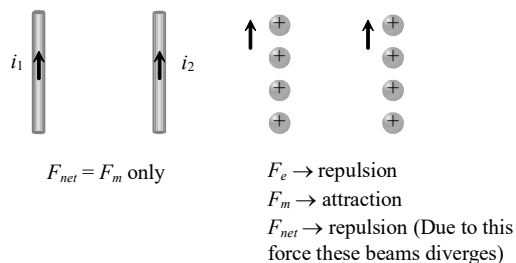
SHM and it's time period is given by $T = 2\pi\sqrt{\frac{h}{g}}$.



21. In the previous case if direction of current in movable wire is reversed then it's instantaneous acceleration produced is $2g \downarrow$.

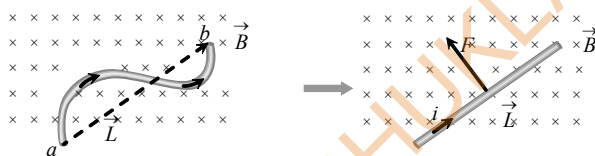
22. Electric force is an absolute concept while magnetic force is a relative concept for an observer.

23. The nature of force between two parallel charge beams is decided by electric force, as it is dominator. The nature of force between two parallel current carrying wires is decided by magnetic force.



24. If a straight current carrying wire is placed along the axis of a current carrying coil then it will not experience magnetic force because magnetic field produced by the coil is parallel to the wire.

25. The force acting on a curved wire joining points a and b as shown in the figure is the same as that on a straight wire joining these points. It is given by the expression $\vec{F} = i \vec{L} \times \vec{B}$



26. If a current carrying conductor AB is placed transverse to a long current carrying conductor as shown then force experienced by wire AB $F = \frac{\mu_0 i_1 i_2}{2\pi} \log_e \left(\frac{x+l}{x} \right)$.

